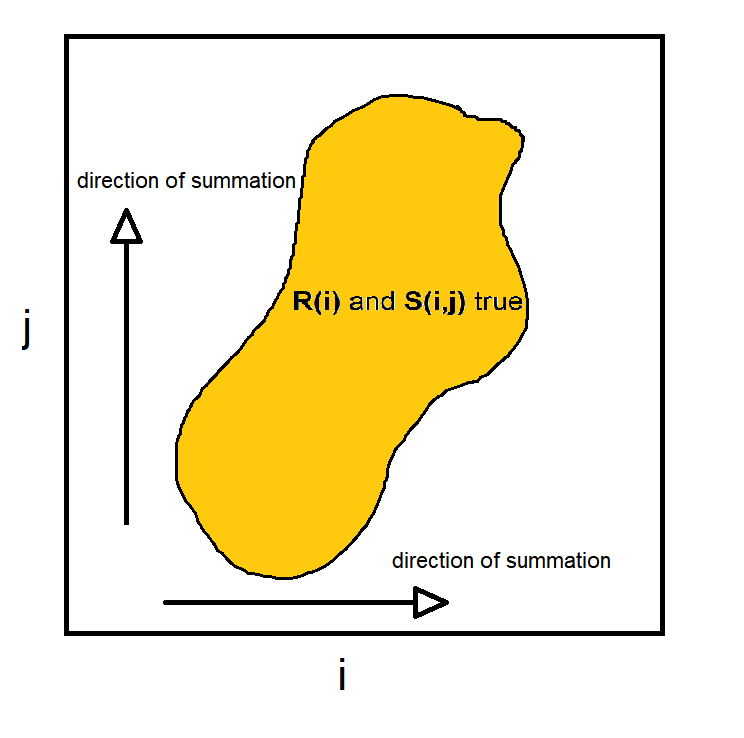
1.2.3 Exercise 15 – interchanging order of summation

Interchange the order of

for which *S* depends on both i and j.

Knuth says that, in principle, it is always possible to perform an interchange of order through the relation

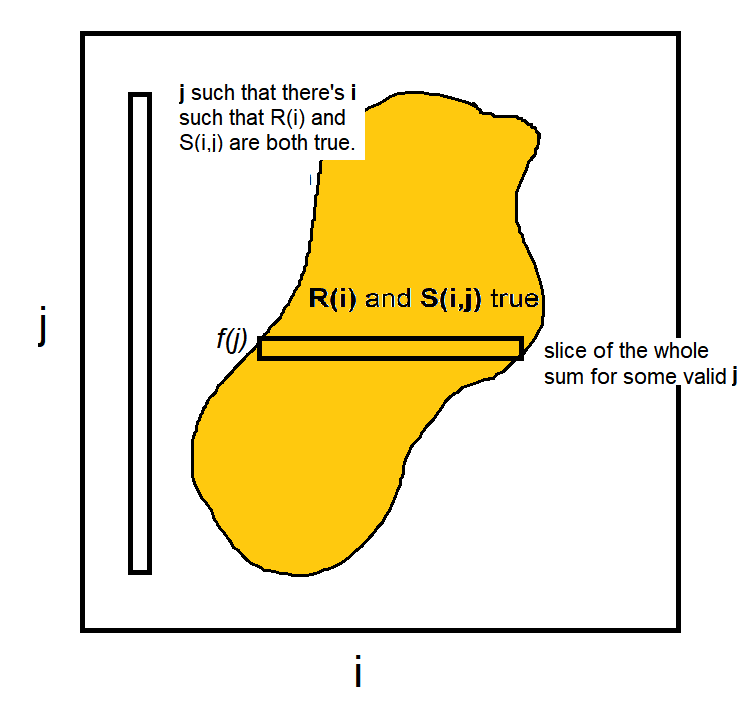
Let’s first investigate the origin of this relation. Bellow is a graphical representation of the full domain of the sum, that can be taken also as infinite in **i** and **j** (have in mind, as Knuth advances, this rule doesn’t stand necessarily for non-convergent infinite sums).



Let’s put the sum together in its more general form (still preserving the directions of summation):

From there it’s possible to break it down again in some number of sub-sums, but now in terms of **j**.

represents a slice of the whole domain. The whole sum then is the sum of all slices for all possible values of **j** such that there’s an **i** such that are true”.



As the first statement only involves the existence of i, it won’t depend on any specific value of it, while the second statement will likely depend on both i and j. Hence the inversion is achieved by this relation.

When the statement “ **are true**” is subjected to the statement “there’s a value of **i**” it will be modified and yield all the conditions in which that statement is true, i.e., in this case, *all the values of* ***j*** *such that there’s an* ***i*** *such that* ***are true.***

Let’s finally interchange the order of .

In this case, using brackets notation, .

So[there’s an i such that R(i) & S(i,j) are true] = ,

and [R(i) and S(i,j) are true] = .

Hence